Exact Soliton Solutions to an Averaged Dispersion-managed Fiber System Equation

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By introducing a set of ordinary differential equations which possess q-deformed hyperbolic function solutions, and a new ansatz, a method is developed for constructing a series of exact analytical solutions of some nonlinear evolution equations. The proposed method is more powerful than various tanh methods, the $\sec_q$-$\tanh_q$-method, generalized hyperbolic-function method, generalized Riccati equation expansion method, generalized projective Riccati equations method and other sophisticated methods. As an application of the method, an averaged dispersion-managed (DM) fiber system equation, which governs the dynamics of the core of the DM soliton, is chosen to illustrate the method. With the help of symbolic computation, rich new soliton solutions are obtained. From these solutions, some previously known solutions obtained by some authors can be recovered by means of some suitable choices of the arbitrary functions and arbitrary constants. Further, the soliton propagation and solitons interaction scenario are discussed and simulated by computer.

Key words: Nonlinear Schrödinger Equation; Dispersion-managed Solitons; Soliton Propagation; Solitons Interaction; Symbolic Computation.

1. Introduction

The dispersion-managed (DM) fiber system has paved a new way to increase the transmitting capacity of optical fiber links [1 – 4]. Basically, the dispersion-management technique utilizes a fiber transmission line with a periodic dispersion map, such that each period is built up by two types of fibers, generally with different lengths and opposite group-velocity dispersion (GVD). Because of the periodic splicing of anomalous and normal dispersion fibers, there is an abrupt discontinuity in the GVD of the DM fiber system. It is very difficult to analytically handle the DM fiber system equation. To analytically describe the evolution of the parameters of the DM solitons, the variational principle is widely used with the help of a Gaussian ansatz [1]. Based on the exact solution of the variational equations, very recently analytical methods have been reported for designing the dispersion map of the DM fiber systems [5, 6]. All these techniques are fundamentally based on the feature that most of the time during the periodic evolution of the DM soliton, the core is very close to a Gaussian shape [7, 8].

Hasegawa et al. [9 – 11] developed a different kind of approach to study the properties of the core of the DM solitons. In that approach they considered the lossless DM fiber system, and after removing the fast varying chirp part of the DM soliton, they derived the averaged DM fiber system equation which governs the dynamics of the core of the DM solitons. Recently, Xu et al. derived some exact solutions for the core of the DM solitons by use of the Darboux transformation and an ansatz [12 – 13] and discussed the interaction between neighboring dark solitons.

In this work, based on a system of ordinary differential equations (ODEs) which possesses the q-deformed hyperbolic function solutions introduced by Arai [14, 15] and various tanh methods [16 – 23], the generalized hyperbolic-function method [24 – 25], the generalized projective Riccati equations method [26], the $\sec_q$-$\tanh_q$-method [27], the generalized Riccardi equation expansion method [30] and other sophisticated methods [9 – 13], we will propose a new method for constructing exact solutions for some nonlinear evolution equations (NEEs). Then we choose the averaged DM fiber system equation [9 – 12]

$$i\frac{\partial q}{\partial z}+\alpha_1\frac{\partial^2 q}{\partial t^2}+\alpha_2|q|^2q=\beta_1(z)t^2q-i\beta_2(z)q \quad (1)$$

to illustrate the method. α_1 and α_2 are arbitrary constants, and $\beta_1(z)$ and $\beta_2(z)$ are the quadratic phase chirp coefficient and the loss (gain) coefficient, respectively. A special case of (1), i.e., without the residual loss (gain) term $\beta_2(z)$ and $\beta_1(z) = \text{const}$, has been studied in [10, 11], where nonlinear compression of chirped solitary waves and quasisoliton propagation in a DM optical fiber have been discussed. In [12], Xu et al. derived the exact soliton solutions of (1) with $\beta_1(z)$ and $\beta_2(z) = \text{const}$ by using the Darboux transformation, and derived a dark soliton solution of (1) by an ansatz. At the same time, they discussed the interaction scenario between two neighboring solitons. Equation (1) derives from the nonlinear Schrödinger equation (NLSE)

$$\frac{\partial u}{\partial z} = \frac{id(z)}{2}\frac{\partial^2 u}{\partial T^2} + i|u|^2 u - \Gamma u, \tag{2}$$

which governs the dynamics of DM fiber systems, where u is the envelope of the axial electric field, $\mathrm{d}(z)$ the periodically variety GVD parameter representing anomalous and normal dispersions, and Γ the loss (gain) coefficient. (See [9 – 13] for details).

The rest of this paper is organized as follows. In Sect. 2, a method for constructing exact analytical solutions of NEEs is established. In Sect. 3, a rich variety of exact solutions of (1) is obtained by the method, and the main features of various analytical solutions obtained are investigated by using direct computer simulations. Section 4 is a short summary and discussion.

2. Method

In [14, 15] Arai introduced the q-deformed hyperbolic functions

$$\sinh_q \xi = \frac{e^{\xi} - qe^{-\xi}}{2}, \quad \cosh_q \xi = \frac{e^{\xi} + qe^{-\xi}}{2}, \tag{3}$$

$$\tanh_q \xi = \frac{\sinh_q \xi}{\cosh_q \xi}, \quad \mathrm{sech}_q \xi = \frac{1}{\cosh_q \xi}, \quad \xi \in \mathbf{C}, \tag{4}$$

where $q > 0$ is a deformation parameter. Note that, if $q \neq 1$, then $\sinh_q \xi$ is not odd and $\cosh_q \xi$ is not even:

$$\begin{aligned} &\sinh_q(-\xi) = -q \sinh_{1/q} \xi, \\ &\cosh_q(-\xi) = q \cosh_{1/q} \xi, \quad \xi \in \mathbf{C}. \end{aligned} \tag{5}$$

The following formulae can be easily proven:

$$(\sinh_q \xi)' = \cosh_q \xi, \tag{6}$$

$$(\cosh_q \xi)' = \sinh_q \xi, \tag{7}$$

$$\cosh_q^2 \xi - \sinh_q^2 \xi = q, \tag{8}$$

$$(\tanh_q \xi)' = q\,\mathrm{sech}_q^2 \xi, \tag{9}$$

$$(\mathrm{sech}_q \xi)' = -(\tanh_q \xi)(\mathrm{sech}_q \xi), \tag{10}$$

$$\tanh_q^2 \xi = 1 - q\,\mathrm{sech}_q^2 \xi. \tag{11}$$

Now, using the above q-deformed hyperbolic functions and the ideas in [16 – 31], we introduce the following system of ODEs:

$$\begin{aligned} \frac{\mathrm{d}\sigma(\xi)}{\mathrm{d}\xi} &= -\sigma(\xi)\tau(\xi), \\ \frac{\mathrm{d}\tau(\xi)}{\mathrm{d}\xi} &= \mu\sigma(\xi) + (q - \mu^2)\sigma^2(\xi), \end{aligned} \tag{12}$$

where

$$\tau^2(\xi) = 1 - 2\mu\sigma(\xi) + (\mu^2 - q)\sigma^2(\xi). \tag{13}$$

ξ is a variable and $\mu, q > 0$ ($q \neq \mu^2$) are constants.

We know that (12) – (13) have the solutions

$$\sigma(\xi) = \frac{1}{\mu + \cosh_q \xi}, \quad \tau(\xi) = \frac{\sinh_q \xi}{\mu + \cosh_q \xi}, \tag{14}$$

where the variable ξ is complex, so that the two functions can be trigonometric or hyperbolic. For simplicity, in this paper, we only consider the case of hyperbolic functions.

Now we outline our proposed method as follows: Given nonlinear NEEs with, say, two variables z and t:

$$p(u_t, u_z, u_{zt}, u_{tt}, u_{zz}, \cdots) = 0. \tag{15}$$

Step 1. We assume that the solutions of (15) are

$$u(z,t) = a_0 + \sum_{i=1}^{m} \sigma^{i-1}(\xi)[a_i\sigma(\xi) + b_i\tau(\xi)], \tag{16}$$

where $a_0 = a_0(z,t)$, $a_i = a_i(z,t)$, $b_i = b_i(z,t)$, $(i = 1, \cdots, m)$, $\xi = \xi(z,t)$ are all differentiable functions of $\{z,t\}$, $\sigma(\xi)$ and $\tau(\xi)$ satisfy (12) – (13). The parameter m can be determined by balancing the highest order derivative term and the nonlinear terms in (15). m is usually a positive integer. If not, some proper transformation $u(z,t) \to u^m(z,t)$ may be in order to satisfy this requirement.

Step 2. Substituting (16) along with (12) and (13) into (15), we can obtain a set of algebraic polynomials for $\sigma^i(\xi)\tau^j(\xi)(i=0,1,\cdots;\;\; j=0,1)$. Setting the coefficients of the terms $\tau^i(\xi)\sigma^j(\xi)$ to zero, we get a system of over-determined PDEs (or ODEs) with respect to unknown functions $\{a_0, a_i, b_i\ (i=1,\cdots,m), \xi\}$.

Step 3. Solving the over-determined PDEs (or ODEs) system by use of the symbolic computation system *Maple*, we would end up with the explicit expressions for μ, a_0, a_i, b_i ($i=1,\cdots,m$) and ξ or the constraints among them.

Step 4. Thus, according to (14) and (16) and the conclusions in **Step 3** we can obtain many families of exact solutions for (15).

The method proposed is more general than the methods in [16–31]. First, compared with various tanh method [16–23], the projective Riccati equations method [28,29] and $\sec_q$-$\tanh_q$-method [27], the restriction on $\xi(z,t)$ as merely a linear function $\{z,t\}$ and the restriction on the coefficients a_0, a_i, b_i ($i=1,\cdots,m$) as constants are removed. Second, the generalized tanh method [24,25], the generalized hyperbolic-function method [30,31] and the generalized projective Riccati equation methods [26] can be recovered by choosing specific parameters: a_0, a_i, b_i ($i=1,\cdots,m$), $\xi(z,t)$, μ and q.

3. Exact Soliton Solutions and Computer Simulations

We now investigate (1) with our algorithm. In order to obtain some exact solutions of (1), we first analyze (1) by separating $q(z,t)$ into the amplitude function $A(z,t)$ and the phase function $\phi(z,t)$ as

$$q(z,t)=A(z,t)\exp[i\phi(z,t)]. \tag{17}$$

Then, substituting (17) into (1) and setting the real and imaginary parts of the resulting equation equal to zero, we obtain the following set of PDEs

$$-A\phi_z+\alpha_1(A_{tt}-A\phi_t^2)+\alpha_2A^3-\beta_1(z)t^2A=0, \tag{18}$$

$$A_z+\alpha_1(2A_t\phi_t+A\phi_{tt})+\beta_2(z)A=0. \tag{19}$$

According to the method described in Sect. 2, we assume the solutions of (18)–(19) in the following special forms

$$\begin{aligned}&A(z,t)=a_0(z)+a_1(z)\sigma(\xi)+b_1(z)\tau(\xi),\\ &\xi=\Omega(z)t+\Gamma(z),\end{aligned} \tag{20}$$

$$\phi(z,t)=\lambda_2(z)t^2+\lambda_1(z)t+\lambda_0(z), \tag{21}$$

where $a_0(z)$, $a_1(z)$, $b_1(z)$, $\Omega(z)$, $\Gamma(z)$, $\lambda_2(z)$, $\lambda_1(z)$ and $\lambda_0(z)$ are all differentiable functions of z to be determined, and $\tau(\xi)$ and $\sigma(\xi)$ satisfy (12)–(13).

Substituting (12), (13), (20) and (21) into (18)–(19), collecting the coefficients of the polynomials of $\tau(\xi)$, $\sigma(\xi)$ and t of the resulting system, then setting each coefficients to zero, we obtain an over-determined ODE system, which consists of 21 ODEs, with respect to the differentiable functions $\{a_0(z),\ a_1(z),\ b_1(z),\ \Gamma(z),\ \Omega(z),\lambda_2(z),\ \lambda_1(z),\ \lambda_0(z),\ \beta_1(z),\ \beta_2(z)\}$ and undetermined constants $\{\mu,q,\alpha_1,\alpha_2\}$. For simplicity, we do not list them in the paper. Solving the ODE system by means of *Maple*, we obtained the following results.

Case 1.

$$\begin{aligned}&a_0(z)=b_1(z)=\mu=0,\;\; \beta_2=2\alpha_1\lambda_2(z),\;\; a_1=C_1\exp\Big[-4\alpha_1\int\lambda_2(z)\mathrm{d}z\Big],\\ &\Omega(z)=\pm\frac{\sqrt{2}\sqrt{q\alpha_1\alpha_2}\exp\big[-4\alpha_1\int\lambda_2(z)\mathrm{d}z\big]C_1}{2q\alpha_1},\;\; \lambda_1(z)=C_3\exp\Big[-4\alpha_1\int\lambda_2(z)\mathrm{d}z\Big],\\ &\lambda_0(z)=\frac{\int\exp\big[-8\alpha_1\int\lambda_2(z)\mathrm{d}z\big]\mathrm{d}z(\alpha_2{C_1}^2-2q\alpha_1{C_3}^2)}{2q}+C_5,\\ &\Gamma(z)=\mp\frac{\sqrt{2}\sqrt{q\alpha_1\alpha_2}C_1C_3\int\exp\big[-8\alpha_1\int\lambda_2(z)\mathrm{d}z\big]\mathrm{d}z}{q}+C_6,\;\; \beta_1(z)=-4\alpha_1\lambda_2^2(z)-\lambda_2'(z),\end{aligned} \tag{22}$$

where $C_1,C_3,C_5,C_6,\alpha_1,\alpha_2,q$ are arbitrary constants and $\lambda_2(z)$ is an arbitrary function.

Case 2.

$$a_0(z)=a_1(z)=\mu=0,\;\; \Omega(z)=-\frac{\sqrt{2}\sqrt{-\alpha_1\alpha_2}\exp\big[-4\alpha_1\int\lambda_2(z)\mathrm{d}z\big]C_1}{2\alpha_1},$$

$$\Gamma(z) = \sqrt{2}\sqrt{-\alpha_1\alpha_2}C_1C_3\int \exp\Big[-8\alpha_1\int\lambda_2(z)\mathrm{dz}\Big]\mathrm{dz}+C_6,\;\; b_1(z)=C_1\exp\Big[-4\alpha_1\int\lambda_2(z)\mathrm{dz}\Big],$$
$$\lambda_0(z)=\int\exp\Big[-8\alpha_1\int\lambda_2(z)\mathrm{dz}\Big]\mathrm{dz}(\alpha_2{C_1}^2-\alpha_1{C_3}^2)+C_5,\;\; \beta_2(z)=2\alpha_1\lambda_2(z),$$
$$\beta_1(z)=-4\alpha_1\lambda_2^2(z)-\lambda_2'(z)),\;\; \lambda_1(z)=C_3\exp\Big[-4\alpha_1\int\lambda_2(z)\mathrm{dz}\Big], \tag{23}$$

where $C_1, C_3, C_5, C_6, \alpha_1, \alpha_2, q$ are arbitrary constants and $\lambda_2(z)$ is an arbitrary function.

Case 3.

$$a_0(z)=0,\;\; \Gamma(z)=2\sqrt{2}\sqrt{-\alpha_1\alpha_2}C_1C_5\int\exp\Big[-8\alpha_1\int\lambda_2(z)\mathrm{dz}\Big]\mathrm{dz}+C_9,$$
$$a_1(z)=-\sqrt{\mu^2-q}\exp\Big[-4\alpha_1\int\lambda_2(z)\mathrm{dz}\Big]C_1,\;\; b_1(z)=C_1\exp\Big[-4\alpha_1\int\lambda_2(z)\mathrm{dz}\Big],$$
$$\lambda_1(z)=C_5e^{-4\alpha_1\int\lambda_2(z)\mathrm{dz}},\;\; \beta_2(z)=2\lambda_2(z)\alpha_1,\;\; \beta_1(z)=-\frac{d}{\mathrm{dz}}\lambda_2(z)-4\alpha_1(\lambda_2(z))^2,$$
$$\lambda_0(z)=\int\exp\Big[-8\alpha_1\int\lambda_2(z)\mathrm{dz}\Big]\mathrm{dz}(\alpha_2{C_1}^2-\alpha_1{C_5}^2)+C_{10},\;\; \Omega(z)=-\frac{\sqrt{2}\sqrt{-\alpha_1\alpha_2}e^{-4\alpha_1\int\lambda_2(z)\mathrm{dz}}C_1}{\alpha_1}, \tag{24}$$

where $C_1, C_5, C_9, C_{10}, \alpha_1, \alpha_2, q$ and μ are arbitrary constants and $\lambda_2(z)$ is an arbitrary function.

Therefore from (14), (17), (20), (21) and (22)–(24) three families of exact solutions of (1) can be derived as follows:

Family 1.

$$q_1=C_1\theta\,\mathrm{sech}_q(\xi)\exp\Big\{\mathbf{i}\Big[\lambda_2(z)t^2+\lambda_1(z)t+\lambda_0(z)\Big]\Big\}, \tag{25}$$

where

$$\theta=\exp\Big[-4\alpha_1\int\lambda_2(z)\mathrm{dz}\Big],\;\; \xi=\pm\frac{\sqrt{2\alpha_1\alpha_2q}C_1\theta}{2\alpha_1q}t\mp\frac{\sqrt{2\alpha_1\alpha_2q}C_1C_3\int\theta^2\mathrm{dz}}{q}+C_6,\;\; \lambda_2(z)=\frac{\beta_2(z)}{2\alpha_1},$$
$$\lambda_1(z)=C_3\theta,\;\; \lambda_0(z)=\frac{\int\theta^2\mathrm{dz}(\alpha_2C_1^2-2\alpha_1C_3^2q)}{q}+C_5,\;\; \beta_1(z)=-4\,\alpha_1\lambda_2^2(z)-\lambda_2'(z).$$

Family 2.

$$q_2=C_1\theta\tanh_q(\xi)\exp\Big\{\mathbf{i}\Big[\lambda_2(z)t^2+\lambda_1(z)t+\lambda_0(z)\Big]\Big\}, \tag{26}$$

where

$$\theta=\exp\Big[-4\alpha_1\int\lambda_2(z)\mathrm{dz}\Big],\;\; \xi=\pm\frac{\sqrt{-2\alpha_1\alpha_2}C_1\theta}{2\alpha_1}t\mp\sqrt{-2\alpha_1\alpha_2}C_1C_3\int\theta^2\mathrm{dz}+C_6,\;\; \lambda_2(z)=\frac{\beta_2(z)}{2\alpha_1},$$
$$\lambda_1(z)=C_3\theta,\;\; \lambda_0(z)=\int\theta^2\mathrm{dz}(\alpha_2C_1^2-2\alpha_1C_3^2)+C_5,\;\; \beta_1(z)=-4\,\alpha_1\lambda_2^2(z)-\lambda_2'(z).$$

Family 3.

$$q_3=C_1\theta\left[\frac{-\sqrt{\mu^2-q}}{\mu+\cosh_q(\xi)}\pm\frac{\sinh_q(\xi)}{\mu+\cosh_q(\xi)}\right]\exp\Big\{\mathbf{i}\Big[\lambda_2(z)t^2+\lambda_1(z)t+\lambda_0(z)\Big]\Big\}, \tag{27}$$

where

$$\theta = \exp\left[-4\alpha_1 \int \lambda_2(z)\mathrm{d}z\right],\ \ \xi = \pm\frac{\sqrt{-2\alpha_1\alpha_2}C_1\theta}{\alpha_1}t \mp 2\sqrt{-2\alpha_1\alpha_2}C_1C_5\int\theta^2\mathrm{d}z + C_9,\ \ \lambda_2(z) = \frac{\beta_2(z)}{2\alpha_1},$$

$$\lambda_1(z) = C_5\theta,\ \ \lambda_0(z) = \int\theta^2\mathrm{d}z(\alpha_2C_1^2 - 2\alpha_1C_5^2) + C_{10},\ \ \beta_1(z) = -4\,\alpha_1\lambda_2^2(z) - \lambda_2'(z).$$

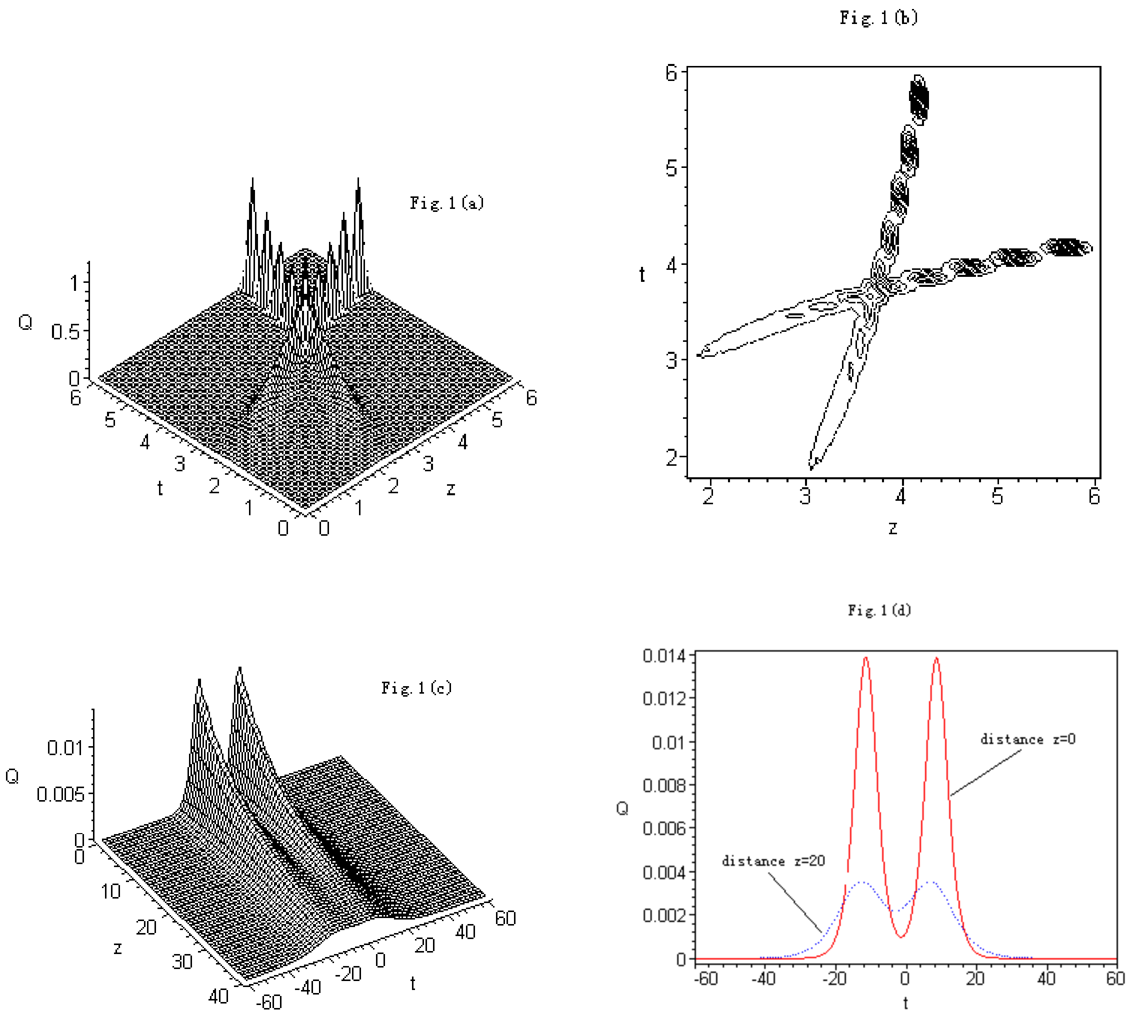

Figure 1. Fig. 1 a, b) describe the interaction of two exponentially growing bright solitons with the scenario given by (25). Initial conditions: $\alpha_1 = 0.8, \alpha_2 = 1, \lambda_2(z) = -0.3, C_1 = C_3 = 0.02, C_6 = 0$; Fig. 1 c) shows the interaction of two equal amplitude pulses with the initial pulse separation equal to 20 with $\theta = 1/(2z + 2\sin(z)\cos(z) + 60), \alpha_1 = 0.5, \alpha_2 = 4, C_1 = 10, C_3 = C_6 = 0$; Fig. 1(d) depicts the pulse shapes of two solitons under the conditions of Figure 1 c).

It is easy to see that, when setting $\lambda_2(z) = \text{const}$, the solutions (20), (33) and (34) obtained in [12] can be reproduced by our solution (25) and (26), respectively. To the best of our knowledge, the solutions (25), (26) with $\lambda_2(z) \neq \text{const}$, and (27) have not been reported earlier.

In order to understand the significance of these solutions expressed by (25)–(27), the main soliton fea-

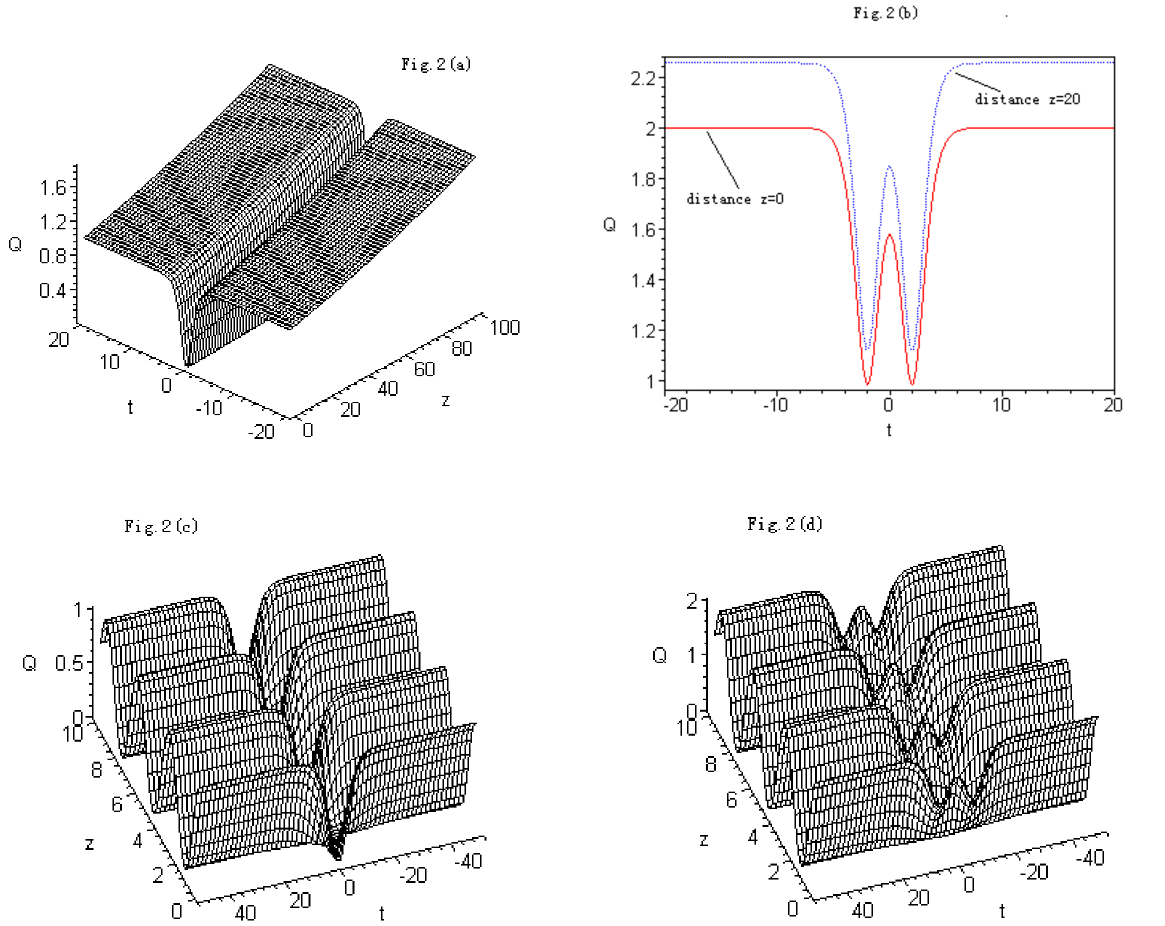

Figure 2. Fig. 2(a) depicts the evolution of an exponentially increasing dark soliton (26) after choosing $\alpha_1 = -0.5, \alpha_2 = 1, \lambda_2(z) = 0.0015, C_1 = 1, C_3 = 3, C_6 = 0$; Pulse shapes of a pair of dark solitons are given by (26), where the parameters are the same as in Fig. 2(a) and the initial separation is equal to 4; Dark soliton propagation and the two dark solitons interaction scenario with the initial pulse separation equal to 12 are simulated in Figs. 2(c – d), where the parameters are: $\theta = \cos(z), \alpha_1 = 1, \alpha_2 = 0.10, C_1 = 1, C_3 = C_6 = 0$.

tures of them are investigated by using direct computer simulations with the accuracy as high as 10^{-9}. For simplicity, we only consider the case of $q = 1$.

As shown in Fig. 1 a, b), the height of the bright soliton increases exponentially as a function of z due to the exponentially increasing nature of $\theta = \exp[-4\alpha_1 \int \lambda_2(z)\mathrm{d}z]$ when $\lambda_2(z) = \text{const} < 0$, $\alpha_1 > 0$ (Notice: in this case and in the following cases the integration constants are taken to be zero) and its width gets compressed during its propagation. At the same time, we can see that the interaction is elastic by having a look. The interaction between neighboring bright solitons is given in Figure 1 c, d). Figure 1 d) shows the pulse shape of the output pulse when the initial soliton separation is equal to 20 after it propagations a distance of $z = 20$ in a fiber. As shown in Fig. 1 d), as the pulse travels further down the fiber, the separation between two solitons goes on increasing. From Figs. 2 a, b) and 3 a, b) we can derive that the cases of dark solitons (26) and (27) are similar to the case of solution (25). Furthermore we also investigate the dark soliton propagation and the two neighboring dark solitons interaction given by (26) and (27). Because the fundamental set of DM solutions can be expressed in trigonometric and hyperbolic functions, we assume that θ is a periodically varying control function: $\theta = \cos(z)$or $\theta =$

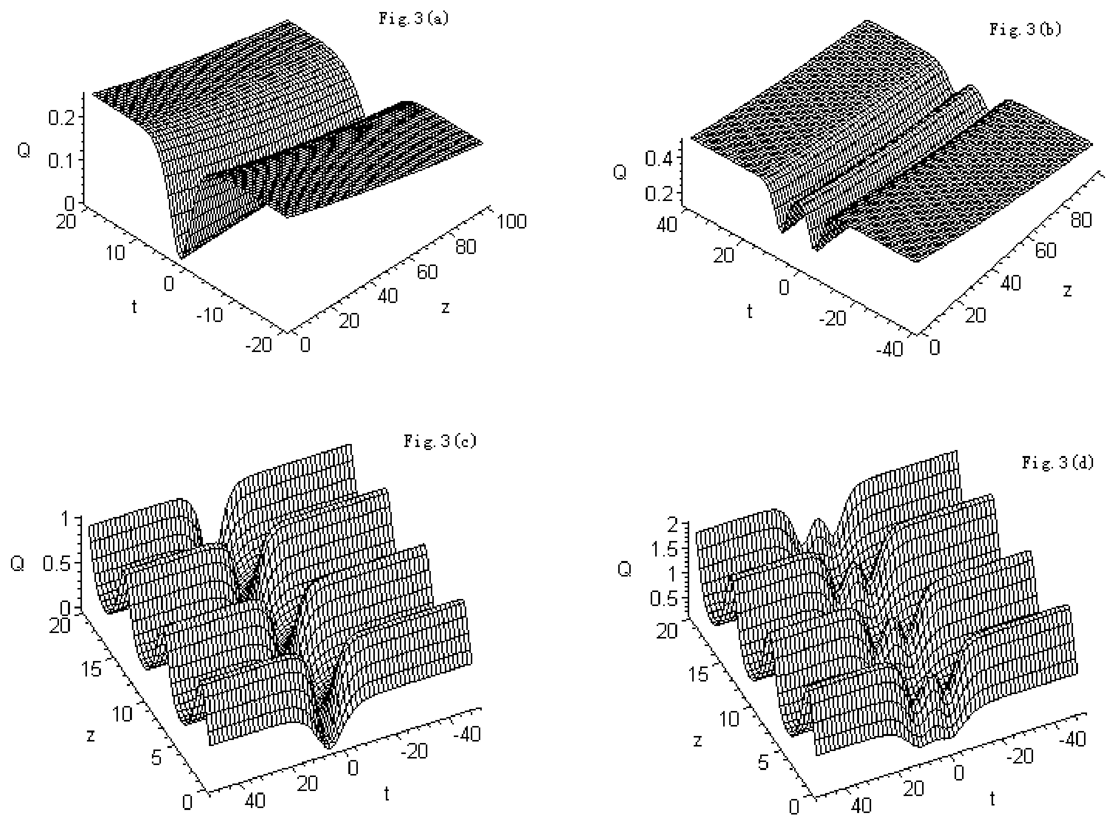

Figure 3. Figs. 3(a – b) depicts the propagation scenario of exponentially decreasing dark solitons given by (27) and their interaction scenario with initial pulse separation equal to 12. Input conditions: $\mu = 2, \alpha_1 = -0.5, \alpha_2 = 0.5, \lambda_2(z) = -0.0015, C_1 == 0.5, C_5 = C_9 = 0$; Figs. 3(c – d) depict the dark soliton propagation scenario and two solitons interaction scenario with the initial pulse separation equal to 12, where the parameters are as follows: $\mu = 2, \theta = 1/(-\sin(z)+3), \alpha_1 = -1, \alpha_2 = 0.10, C_1 = 2, C_5 = C_9 = 0$.

$1/(-\sin(z)+3)$, i. e., $\lambda_2(z) = \tan(z)/(4\alpha_1)$ or $\lambda_2 = -1/[4\alpha_1(-\sin(z)+3)]$. From the periodicily property of θ, we can derive that $\beta_1(z)$ and $\beta_2(z)$ are periodic functions. As is shown in Figs. 2 c, d) and 3 c, d), the dark soliton propagation and the two neighboring dark solitons interaction possess some periodic properties.

4. Summary and Discussion

Based on various tanh methods, the generalized hyperbolic-function method, the generalized Riccati equation expansion method, the generalized projective Riccati equation method, the $\sec_q$-$\tanh_q$-method and other sophisticated methods, and a new system of ODEs, a method is developed for constructing a series of exact analytical solutions of some NEEs. As an application of the method, an averaged dispersion-managed (DM) fiber system equation, which governs the dynamics of the core of the DM soltions, is chosen to illustrate the method. With the help of symbolic computation, rich new soliton solutions are obtained. From our solutions, some previously known solutions obtained by other authors can be recovered by means of some suitable choices of the arbitrary functions and arbitrary constants. Further, the soliton propagation and solitons interaction scenario are discussed and simulated by computer. The results obtained in the paper are of general physics interest and should be readily verified experimentally. The method proposed here can be applied to other NEEs and coupled ones.

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